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Predatory Dynamics of Language Ecology: Evolutionary Competition between Dominant and Non-Dominant Languages Based on a Lotka-Volterra Model

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ABSTRACT

This study introduces the ecological Lotka-Volterra model into the analysis of language competition and develops a random dynamic framework to capture interactions between dominant and non-dominant languages. Through a system of coupled differential equations, it examines how regional language ecosystems evolve under the global diffusion of a shared communicative language. The results reveal some key dynamic mechanisms. In the natural evolution of a bilingual society, the system tends to form self-organized equilibrium states, marked by periodic coexistence. When social fluctuations intensify systemic phase transitions, the actual usage level of the dominant language may decline unexpectedly. This decline accelerates the process of language assimilation. In addition, chaotic oscillations may emerge if there are mismatches between language promotion policies and the system's intrinsic dynamics. This leads to nonlinear deviations between policy targets and actual dominant language coverage. Based on these findings, the study suggests that effective coordination between language dissemination and diversity protection requires a dynamic adaptive governance. Such a strategy should involve three key actions. First, define critical disturbance thresholds to prevent abrupt declines in dominant language vitality; Second, optimize the timing and frequency of policy interventions to stabilize language coverage; Third, utilize the

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cultural resilience of non-dominant languages to strengthen intergenerational transmission within specific social domains. Overall, this framework provides a cross-disciplinary reference for analyzing and governing multilingual ecosystems.

Keywords: Lotka-Volterra Model; Language Competition; Stochastic Dynamics; Language Diversity Protection; Self-Organizing Equilibrium

1. Introduction

As an indispensable part of the heritage of human civilization, linguistic diversity is rapidly decreasing, and it has become a complex social and ecological problem against the backdrop of globalization. In 2023, according to official statistics, UNESCO (United Nations Educational, Scientific and Cultural Organization) announced that approximately 40% (more than 2600 languages) of the world's approximately 6700 languages were facing a crisis of survival, ranging from reduced use to functional disappearance. This critical percentage marks the systematic collapse of civilized memory. The dynamic origin of this crisis is deeply rooted in the asymmetric competition structure of the regional language ecosystem: in a specific socio-geographical unit, there is a continuous interaction mechanism between the dominant languages (including global languages, such as English and Mandarin, and powerful regional languages, such as Spanish and Hindi), which occupies a dominant position through institutional strengthening, while the non-dominant languages (including dialects, minority languages, indigenous languages and small local languages) are marginalized. It is particularly noteworthy that with the expansion of the cross-border trade network promoted by the Belt and Road Initiative, the global network influence of Confucius Institutes in the field of education and the cultural communication matrix created by the growing content of Chinese numerals, the structural leap of Chinese international influence is changing the traditional English language power model. By 2024, the number of people learning Chinese worldwide will exceed 200 million, and 85 countries will incorporate Chinese into their national education systems. The number of HSK candidates is expected to grow by an average of 20% annually. This growth lends new geopolitical significance to changes in global language competition driven by Chinese communication. Within this context, the dominant language expands through a multi-layered institutionalization. A crucial channel is administrative penetration via the

education system. In public education, the dominant language serves both as a compulsory subject and a medium of instruction. It is adjustments in bilingual education policies and language classification systems within China's ethnic minority regions that directly influence the transmission of non-dominant languages across generations. Currently, the administrative system's mandatory replacement involves the use of the dominant language in legislation, the judiciary, and public services, such as the exclusive use of Indian English in the bureaucratic system. This reduces the space for non-standard languages in public affairs. Economic opportunity manipulation involves higher premiums for dominant language skills in the labor market (Chinese companies in Southeast Asian countries prefer employees who speak Chinese), leading people to adjust their language choice strategies based on survival rationality. Faced with this systemic pressure, non-dominant languages draw on their substrates of cultural resistance to develop adaptive resistance in a limited domain. The living legacy of intangible cultural heritage, such as oral epics, ritual songs, and traditional ecological knowledge systems, maintains the spiritual core of the community's cultural identity. Closed religious rituals and specific areas of practice formed by interracial marriage have preserved the total decline of linguistic functions; dialectical tools for protecting the power of digital technology and the social media language revitalization movement are opening up new spaces for virtual spaces. These scattered forms of resistance may slow the assimilation process, but it is difficult to reverse the dominant trend of language expansion in macro-social structures. Particularly serious is the intervention of artificial intelligence technologies, which exacerbates the ecological imbalances of language. AI training data represents more than 80% of the English language, reinforcing the numerical advantage of high-resource languages that lack key technologies such as machine translation and content generation (e.g., data is insufficient for indigenous languages of Africa and Southern Yoruba). Although some countries have launched localized artificial intelligence ini-

tiatives to tackle this problem, data scarcity and technical limitations remain the structural barriers. These constraints restrict the effective support of non-traditional languages. As a result, the language ecosystem reflects deeper tensions caused by linguistic competition. The communicative advantage of a dominant language arises from its functional utility in modernization. Chinese, for example, transforms linguistic competence into economic capital through the “Chinese + professional skills” model. In contrast, the survival of non-dominant languages relies on the lasting value of cultural identity. For this reason, a systematic analysis of the nonlinear interaction between dominant and non-dominant languages is essential. More attention should be placed on changes in competitive intensity caused by the global spread of emerging languages. This kind of analysis is vital for building a bidirectional governance framework that combines language technology demands with the preservation of cultural inheritance. It also guides intervention strategies to address the crisis of civilizational diversity.

However, there are some methodological limitations in sociolinguistic research on language competition dynamics. Existing methods primarily rely on qualitative discussions^[1] or static data statistics^[2], tending to analyze single factors such as policy interventions or identity recognition in isolation. These methods struggle to effectively quantify the complex bidirectional feedback between the two language communities: dominant languages leverage their large user base to create network effects, continuously attracting non-dominant language groups to switch languages; Meanwhile, non-dominant languages achieve localized revival through cultural awareness movements in specific domains, forming a “decay resistance effect” that slows their decline. Due to the lack of systematic modeling of such dynamic interactions, long-term predictions of the effectiveness of language protection policies often miss the mark. For example, the promotion of Mandarin in China has been highly effective, but it has also raised concerns about the vitality of dialects (such as Cantonese and Wu) and the inheritance of some minority languages; In overseas Chinese communities and regions where Mandarin Chinese is popular, the interaction between the spread of Mandarin Chinese and local indigenous languages or community dialects also requires scientific examination (e.g., the coexistence of Mandarin Chinese and local ethnic languages in Southeast Asia). Evidence from

comparative bilingual communities similarly shows that even strong institutional efforts cannot easily offset the structural dominance of majority languages^[3], and the persistence of linguistic hierarchies despite policy support underscores the difficulty of altering long-term dynamics in competitive language ecosystems.

To address these limitations, this study introduces a classic ecological framework of the Lotka-Volterra model^[4, 5] to establish a quantitative framework for language competition dynamics. The applicability of this model rests on three structural similarities between language competition and ecological interaction systems. First, language diffusion follows a density dependence mechanism, with growth rates rising along with existing user populations. Second, nonlinear inhibitory effects show when dominant languages suppress non-dominant ones, producing a saturation effect. Finally, the system tends toward dynamic equilibrium and may exhibit either stable coexistence or oscillatory behaviour. The above Lotka-Volterra model is a valuable tool for analysing language coexistence, replacement, and competition in shared social environments.

The Lotka-Volterra model has shown strong analytical capacity for nonlinear dynamics within complex systems. This strength has supported its wide application in diverse disciplines. It was first proposed independently in the 1920s by Alfred J. Lotka and Vito Volterra. It was originally developed to describe predator and prey population interactions in biological populations, and then the model reveals principles of competitive exclusion and periodic oscillation. The following research has extended its application to economics, sociology, and linguistics, where it has been proven effective in capturing equilibrium relationships within competitive systems.

In terms of foundational theoretical research, Lotka was the first to systematically illuminate the mathematical principles of population dynamics^[4]. Volterra later established a complete system of competition equations through his study of fish populations in the Adriatic Sea^[5]. These pioneering works laid a solid theoretical groundwork for later research on dynamic competition systems. Based on the above, Jones introduced ecological concepts into the study of language change. Through empirical investigations of Welsh-speaking communities, he uncovered the social mechanisms of language shift. This work opened new ways for interdisciplinary

applications of ecological models in linguistics^[3]. Fishman further explored strategies and methods for language preservation^[1]. Crystal emphasized the urgent need to protect linguistic diversity by studying language extinction^[2]. A breakthrough came from Abrams and Strogatz. They applied the Lotka–Volterra model to the quantitative modeling of language extinction processes, thus establishing a new paradigm for formalizing language competition^[6]. Subsequent researchers refined the model. Patriarca and Leppänen, for example, enhanced the competition coefficient to achieve greater descriptive accuracy^[7]. Kandler then incorporated demographic factors to strengthen the model's predictive power and practical relevance^[8]. To address uncertainty-related limitations in traditional formulations, Wu et al. developed a gray Lotka–Volterra model^[9]. Klimenko applied a bounded Lotka–Volterra model to simulate leaping cycles in technological progress and industrial safety, revealing common cyclic mechanisms across complex evolutionary systems^[10]. Zhao Lichun et al. introduced pulse control theory to the regulation of the Lotka–Volterra system, offering new insights into system control^[11]. Gatabazi et al. integrated fractional calculus with grey theory to propose a Fractional Gray Lotka–Volterra Model (FGLVM) for predicting the transaction dynamics of cryptocurrencies such as Bitcoin, Litecoin, and Ripple, which enhanced the forecasting accuracy^[12]. In the context of social systems, Li Mingshan et al. examined information interactions in online social networks using a discrete Lotka–Volterra information diffusion model, contributing to a dynamic understanding of information dissemination processes^[13]. Brunner and Straßegger applied Lotka–Volterra models to analyze the evolution of market shares for renewable energy and its electricity, exploring the relationship between model complexity and susceptibility to policy interventions, thereby providing quantitative tools for energy market trend research^[14]. Tsai et al. quantified the competitive dynamics between fossil fuels and low-carbon energy consumption in the United States, revealing complex and sometimes counterintuitive energy competition patterns^[15], and Deng and Long applied the Lotka–Volterra model to quantify the symbiotic relationships among government, enterprises, and academic institutions within China's regional green innovation ecosystems, a systematic dynamic modeling tool for formulating regional green innovation policies^[16]. Beyond energy and environ-

mental studies, Ivanova et al. introduced the Lotka–Volterra model into economic complexity research by constructing a ternary complexity index, which allowed for a dynamic analysis of innovation ecosystems^[17]. Yang et al. explored the symbiotic interactions between the digital economy and the real economy and then provided theoretical support for the digital economy development^[18]. Mao et al. combined the Logistic model and the Lotka–Volterra model to analyze the evolutionary patterns in the construction robot industry ecosystems^[19]. Wang Z.P. and Wang L.J. further introduced neural network methods into the parameter learning of random Lotka–Volterra systems, which promoted the intelligent model development^[20].

Extensive interdisciplinary applications have shown that the Lotka–Volterra model is widely applicable. It effectively handles systems that are defined by competitive co-existence and feedback regulation. This foundation enables the systematic modelling of language competition dynamics, the quantitative analysis of key driving factors, such as user base, socioeconomic appeal, and cultural heritage intensity. It enables the prediction of long-term evolutionary trends and then forms more precise strategies for language maintenance and diversity protection^[21–25].

The main innovations of this study are: firstly, the first construction of a language competition dynamics model incorporating random disturbances and policy interventions; secondly, the innovative use of the Gaussian truncated moment method to handle system parameters; and thirdly, the proposal of a dynamic intervention mechanism with practical guidance significance. These innovations not only enrich the theoretical framework of the Lotka–Volterra model but also provide new research perspectives and methodological tools for language diversity protection.

2. Language Competition Migration in the Lotka–Volterra Model

2.1. Theoretical Basis of the Model

The Lotka–Volterra model, proposed by biochemist Alfred Lotka^[4] and mathematician Vito Volterra^[5], was first used to describe the dynamic balance between predator and prey populations in ecosystems. The theoretical basis of the model is a mathematical abstraction based on competing

repulsion mechanisms and describing the dynamic evolutionary relationships between two groups using nonlinear differential equations. His contribution to nuclear theory can be summarized in three points. First, a dependent characteristic of population density growth is identified, which assumes that individual species follow the laws of exponential growth under conditions of infinite resources, but are limited by the capacity of the environment and tend toward logical equilibrium of the system. Second, the nonlinear mechanism of mutual exclusion, which highlights competing interactions, usually manifests itself as a predator-prey negative feedback cycle in the system. The expansion of the predator population depends on the depletion of prey resources, which in turn suppresses the reproduction of predators. The third system revealed the stability of dynamic equilibrium with periodic oscillations with specific phase relations without perturbation, cyclicity arising from the inherent properties of the structure of differential equations, rather than random noise. Together, these laws form a paradigm framework for ecological dynamics, whose universality derives from the mathematical characteristics of competitive equilibrium. Modern theoretical development focuses on three directions, breaking the boundaries of the classical model: parameter inversion technology through algorithm optimization to calibrate the inverse interaction coefficient of observational data, to achieve quantitative analysis of competition intensity; Integrated spatial heterogeneity distributes continuous diffusion processes in a grid model, combined with neighbor competition rules, to simulate the impact of habitat fragmentation on population distribution; Several coupling mechanisms extend the main binary systems to multidimensional equations to analyze the disruptive effects of higher-order interactions on stability. At the same time, in response to the simplifications inherent in classical models (linear functional responses that ignore predator saturation effects and deterministic frameworks that do not adapt to environmental randomness), science proposes a two-step correction program: the introduction of a nonlinear function response to more realistically describe the resource consumption process. Stochastic differential equations are used to characterize environmental fluctuations through noise, which improves the predictability of the model.

The present research achieves a key breakthrough through the deep integration of artificial intelligence tech-

nologies. Proxy models built through neural networks can adapt to complex group dynamics. Enhanced learning algorithms improve parameter inversion efficiency and strengthen model robustness under unstable conditions. Although the theoretical assumptions remain relatively simplified, the long-term significance is clear. It offers a unified analytical framework for complex, mutually exclusive systems. By incorporating random process theory, explicit spatial modeling, and multilevel interaction structures, the framework offers deeper insights into ecosystem structures, social system behaviors, and dynamic evolutionary processes.

2.2. Theoretical Transfer of Language Competition

The Lotka-Volterra model is applied to language competition analysis due to a deep structural similarity between ecosystems and sociolinguistic systems. This similarity is based on a three-part correspondence. First, a density-driven growth mechanism. The growth rate of a language user population is tied to its current size, creating a reinforcing cycle. However, this growth is not unlimited and is constrained by environmental carrying capacity, which aligns with the mathematical logic of population density effects in ecosystems. Second, the asymmetric competitive exclusion in resource competition: within limited social resource allocation (e.g., educational opportunities, media coverage, career development), dominant languages compress the living space of non-dominant languages by leveraging institutional advantages (e.g., mandatory official language status) and the accumulation of cultural capital. The intensity of this competition can be quantified by asymmetric competition coefficients, a mechanism isomorphic to that of predator-resource exploiters in ecosystems. Third, the tendency toward steady-state configurations: through the nonlinear interaction of competitiveness parameters (e.g., linguistic utility index, institutional support, intergenerational stability), the language competition system forms steady states such as balanced bilingual coexistence or hierarchical multilingualism. Its dynamic convergence process corresponds to the population equilibrium mechanism in ecosystems. The migratory value of this model lies in the fact that it provides an instrumental basis for determining the competitive intensity of a language, parameters such as competitive factors, internal growth rates, environmental opportunities, etc. Interaction between differ-

ent language groups can be quantified. He also developed a guide to systemic stability, which provides a mathematical basis for predicting long-term trends in language development.

Based on the above isomorphism, a set of coupled differential equations for language competition is established:

$$\frac{dx}{dt} = \alpha_1 x - \beta_1 xy \quad (1a)$$

$$\frac{dy}{dt} = -\alpha_2 y + \beta_2 xy \quad (1b)$$

where $x(t)$ represents the density of mainstream language users, and $y(t)$ represents the density of non-mainstream language users. The parameters are defined as follows:

α_1 : Internal growth rate of the first language: This parameter is the upper limit of the natural capacity of the first language to spread without competition, the intensity of which is determined by the systematic connection of the fundamental elements of society. The institutionalization of penetration into the education system reflects the synergies of linguistic engagement and the strength of teacher accreditation in educational programs from time to time and directly guarantees the effectiveness of intergenerational knowledge transfer channels; The density of traditional content transmission and the effectiveness of digital platform recommendation algorithms for media transmission network penetration, systematically demonstrating the public visibility of the language; The political support mechanism optimizes infrastructure provision through fund allocation, and the three integrated driving forces together form a dynamic foundation for language dissemination.

β_1 : This system quantifies the blocking effect that non-dominant languages have on the expansion of dominant languages, and its core is the result of structural conflicts within the symbolic system. Differences in practical norms within a given cultural scene create symbolic power tensions in the social sphere and expand the psychological boundaries between language groups. Such tensions manifest themselves as acts of cultural identity preservation and directly increase social friction in language transmission. When cognitive differences exceed a critical threshold, micro-interactions significantly increase decision-making delays and reduce nonlinear competition at the macro level.

α_2 : This parameter describes the rate of spontaneous

language decline in a local environment free from external pressure, and its evolutionary dynamics are rooted in the structural vulnerability of the genetic system between generations. Aging, oriented toward the age spectrum of the population, directly leads to a decrease in the genetic density of the ecosystem, and the self-sufficiency of language systems will decline even further if the disintegration of traditional functional areas continues in the process of modernizing society. When these two mechanisms form a positive feedback loop, the decline in language viability is characterized by endogenous acceleration, and the magnitude of speed fluctuations negatively correlates with the structural stability of the community.

β_2 : Cultural resistance is a measure of the systemic repression of the influence of a major language on a non-major language, and its mechanism of action represents the dual structure of the institutional economy. The value system of linguistic capital is reconstructed by the mechanism of market distribution of supply and demand, which has had a negative impact on the labor market for non-linguistic skills. The design structure of the system is based on creating barriers to access to public services and gradually creating barriers to access to public resources. Together, they form structural compression fields in an ecological environment, leading to the degradation of linguistic functions towards modes of survival support.

Note: $\alpha/\beta/\gamma/\delta$ in the initial definition is reconstructed as a two-parameter system ($\alpha_1, \alpha_2, \beta_1, \beta_2$) to ensure the logical consistency of symbols.

To strengthen the interdisciplinary grounding of the model, the core parameters are aligned with the classical dimensions of language vitality as follows^[1]:

α_1 (dominant-language endogenous growth) corresponds to institutional support, educational penetration, and media exposure; α_2 (non-dominant natural decay) reflects Fishman's intergenerational transmission strength and domain functionality decline; β_1 (competitive suppression) corresponds to economic opportunity distribution and administrative exclusivity (e.g., dominant-language replacement in bureaucracy); β_2 (cultural resilience) aligns with identity-based resistance, cultural heritage maintenance, and community clustering (e.g., cultural resilience buffers described in Section 3.2); D (disturbance intensity) is linked to vitality indicators involving population mobility, economic shocks,

and technological accessibility; γ and ω (policy intensity and periodicity) relate to institutional support cycles and policy responsiveness.

The above provides a theoretical foundation. It connects the dynamical model to sociolinguistic evaluation frameworks.

2.3. The Extension Mechanism of the Model

Language ecosystems in the real world are shaped by both social random disturbances and policy interventions. Based on random dynamical system theory, this study introduces two types of extension factors into the baseline model. First, a social random disturbance term, denoted as $\xi(t)$, is introduced. This term follows a Gaussian distribution, $\xi(t) \sim N(0, \sigma^2)$. It represents the impact of sudden, unpredictable social events. Examples include wars, large-scale migration waves, and major economic crises. These events directly affect language-using communities. The variance σ^2 reflects the intensity of social fluctuations, and the social disturbance intensity $D = \sigma$ (standard deviation) is defined, where $D \in [0,1]$ represents the intensity of fluctuations; Second, the policy intervention term $\gamma x \cdot \cos \omega t$ adopts a periodic function form to characterize the intervention effects of language protection policies: γ represents the intensity of policy implementation (such as the intensity of financial investment in bilingual education), and ω reflects the periodic characteristics of policies (such as the angular frequency corresponding to a five-year plan).

The extended non-mainstream language dynamics equation about Equation (1a) is:

$$\frac{dx}{dt} = \alpha_1 x - \beta_1 xy + \xi(t) + \gamma x \cdot \cos \omega t \quad (2)$$

where the standard deviation of $\xi(t)$ is $\sigma^2 = D^2$, with $D \in [0,1]$ representing the dimensionless disturbance intensity; $\gamma > 0$ indicates the strength of language protection policies. Since $\xi(t)$ is a random parameter, statistical methods are applied for analysis, using the moment equations corresponding to Equations (2) and (1b) to calculate the expected values $E[x]$ and $E[y]$ of x and y as analytical data. Using the Gaussian truncated moment method, this paper establishes a system of equations for the first and second moments of x and y .

$$\frac{d}{dt} E[x] = \alpha_1 E[x] - \beta_1 E[xy] + \gamma E[x] \cos(\omega t) \quad (3a)$$

$$\frac{d}{dt} E[y] = -\alpha_2 E[y] + \beta_2 E[xy] \quad (3b)$$

$$\begin{aligned} \frac{d}{dt} E[x^2] = & 2\alpha_1 E[x^2] - 2\beta_1 (E[x^2]E[y]) \\ & - 2E[y]E[x]^2 + 2E[xy]E[x]) \\ & + 2\gamma E[x^2] \cos(\omega t) + D \end{aligned} \quad (3c)$$

$$\begin{aligned} \frac{d}{dt} E[xy] = & \alpha_1 E[xy] - \beta_1 (E[x]E[y^2]) \\ & - 2E[x]E[y]^2 + 2E[xy]E[y]) + \gamma E[xy] \cos(\omega t) \\ & - \alpha_2 E[xy] + \beta_2 (E[x^2]E[y] - 2E[y]E[x]^2) \\ & + 2E[xy]E[x]) \end{aligned} \quad (3d)$$

$$\begin{aligned} \frac{d}{dt} E[y^2] = & -2\alpha_2 E[y^2] + 2\beta_2 (E[x]E[y^2]) \\ & - 2E[x]E[y]^2 + 2E[xy]E[y]) \end{aligned} \quad (3e)$$

2.4. The Steady State Maintenance and Disturbance Response Mechanism of Language Ecology

A language ecosystem in steady state functions as a dissipative structure. The system maintains stability through continuous energy exchange under external perturbations. Under environmental pressure, endogenous regulation allows the system to preserve dynamic equilibrium. This concept is rooted in open-system equilibrium theory and the synergistic coexistence model. The steady state of the language ecosystem is realized through three universal mechanisms. First, dynamic oscillatory equilibrium appears as periodic fluctuations in language usage around a baseline, with amplitudes typically not exceeding 15%. This oscillation reflects a balance between institutional propagation power and cultural carrying capacity. When institutional expansion intensity exceeds the regional population's critical carrying threshold of 50%–60%, a collective rebound mechanism emerges and suppresses further expansion. Second, the cultural carrying capacity threshold provides non-mainstream languages to retain intergenerational transmission resilience at extinction-risk levels of 10%–15%. This mechanism introduces a J-shaped recovery inflection point during the decline. And these mechanisms define the boundary conditions of a bistable attraction basin, within which system trajectories converge toward different stable states depending on disturbance intensity. System resilience and adaptive capacity describe the nonlinear recovery behaviour following disturbance. Their effectiveness relies on energy dissipation efficiency across the social network topology. Communities

with high clustering coefficients (greater than 0.6) leverage small-world effects to transform strong external shocks into weak internal signals. This structure reduces language fluctuation amplitudes by roughly 45%–62%. Cultural resilience reserves further enhance repair efficiency through implicit social contracts, such as mother-tongue transmission within families. Evolutionary plasticity reflects the system's long-term ability to reconstruct equilibrium through technological integration and policy adjustment. Digital technologies, for instance, compress cross-language circulation latency below the critical threshold of 0.2 s. This compression expands the survival space of marginal languages in virtual environments by 13.6%–18.4%. Meanwhile, policy interventions fine-tune language status parameters ($|\Delta S_k| < 0.1$) to regulate phase-space trajectories, increasing the probability of bilingual coexistence by more than 32%. The structural characteristics of the language ecosystem form the physical basis for its steady-state equilibrium. This structure is organized as a nested hierarchy across three levels. At the micro level, individual code-switching operates as a game, where decisions are made dynamically based on cost-benefit optimization. At the meso level, community language networks enable transmission. Highly connected clusters slow the growth of information entropy here. At the macro level, power distribution takes effect. This includes the institutional influence of dominant languages and the resistance mechanisms of marginal languages against cultural seepage. Steady-state conditions can be quantified using multiple indicators. High-resilience states correspond to Shannon diversity indices H_L exceeding 2.5. Functional vitality indices, defined by formal usage rates above 60%, serve as safety thresholds. Intergenerational inheritance strength is indicated by parent-child transmission probabilities above 0.6, while domain penetration rates exceeding 40% in education and media reflect stable system phases. Steady-state maintenance relies on a dual-track coupling mechanism. Endogenous regulatory loops suppress imbalance through negative feedback. When institutional transmissibility expands beyond critical levels, ethnic resilience is activated and dampens acceleration. At extinction-critical points, cultural carrying capacity thresholds initiate intergenerational repair chains and reverse decline gradients. Exogenous regulation builds a buffer layer by coupling policy coordination coefficients ($\gamma_p = 0.18 \pm 0.05$) with cultural resilience gains ($\beta_c = 0.32 \pm 0.03$). Pol-

icy tools in this framework include functional zoning planning to protect language purity in core cultural domains and technology-enabled channels such as language-specific algorithm modules. The synergy between these two types of policy tools tracks enhances system resilience by more than 50%. Disturbance response constitutes the core dynamic process sustaining steady states. Environmental disturbances, such as migration intensity exceeding $D > 0.5$, reconfigure language network topology through population mobility. This reconfiguration induces fluctuations in language usage rates, typically characterized by a decay slope greater than 3.5 units per year. Structural disturbances, including digital divides or abrupt policy changes, weaken system foundations by introducing algorithmic bias or institutional rigidity. The system's response follows a two-stage dissipation path: the resistance response period buffers disturbance energy through redundant channel diversion (shock diffusion in community networks), intergenerational repair initiation (emergency reinforcement of inheritance chains), and functional substitution compensation (temporal displacement of physical decline by digital scenarios). Its core feature is non-linear response near the extinction critical point (usage rate exceeds linear prediction values by 15% against the trend). The adaptive reconstruction phase rebuilds the equilibrium point through policy parameter calibration (progressive correction of S_k), technological interface integration (ecological niche construction of vertical corpora), and community participation enhancement (energy level enhancement of language sanctuary projects), achieving a system deviation convergence rate $>46\%$ and a steady-state retention rate 12% higher than theoretical expectations. The ultimate form of linguistic ecological homeostasis is dissipative equilibrium at the edge of chaos: the system stimulates the emergence of innovation (such as mixed language generation or semantic reconstruction) through energy dissipation (such as cognitive conflict in multilingual learning), while constraining disordered expansion at the boundary of the attractor domain (annihilation critical point), thereby achieving the synergy of diversity preservation and civilizational evolution. The evolution of this equilibrium exhibits a four-stage dynamical process: the periodic oscillations of the equilibrium phase (amplitude $\pm 25\%$, phase difference 150° – 180°) reflect the attractor characteristics of the intrinsic steady state; the amplification effect of institutional rigidity in the perturbation

phase (exponential decay of the dominant language during the window period) exposes the system's Lyapunov instability; the cultural resilience inflection point of the resistance phase (super-threshold repair at the extinction critical point) triggers phase space trajectory folding, reversing the system's entropy increase direction; and the policy-technology synergy of the reconstruction phase (bi-parameter coupling deviation convergence) ultimately drives the system to migrate to a new attractor basin. The evolutionary process shows strong nonlinear characteristics, including sensitivity to minor perturbations, path dependence shaped by historical policies, and abrupt threshold transitions. For system sustainability, three core principles prove essential. The first principle highlights creative transformation of fluctuations to achieve crisis metamorphosis, such as resolving learning conflicts through language hybridization. The second principle focuses on targeted regulation at bifurcation points through threshold management of language status parameters. The third principle stresses adaptive iteration during reconstruction phases, including dynamic optimization of algorithmic modules to enhance system resilience.

2.5. Adaptability Verification Framework for Model Transfer and Cross-Scenario Parameter Calibration

The transfer of the Lotka-Volterra model from biological ecology to linguistic ecology requires the implementation of an adaptive validation and parametric cross-calibration system to overcome the difficulties of generalizing the model and consolidate the theory of scientific migration. Biological predators end competition through the disappearance of individuals, while linguistic skills focus on modifying the linguistic status of users in mutually exclusive biological groups, bilingual ecological breeding groups. While biological competition stems from competition for resources, linguistic competition is a game of power and cultural identity. These significant differences determine that the migration model cannot simply be a formal analogy; it must clearly explain empirical limits and provide parameters of sociocultural significance through scenario calibration.

The fit test is a three-step structure based on microscopic-intermittent kinetic-macroscopic behavioral trends. At the microscopic level, language proficiency parameters (such as the dominant language growth factor α_1 and the

proficiency factor β_1) are not reproduction or resource consumption coefficients in biological models, but a statistical sum of individuals' language choice behaviors in different contexts (family, work, society). The construction of a probability model for individual linguistic decision-making can deduce the functional relationship between macro parameters and the probability of scenario change, the intensity of cultural identity, the subject's growth rate, and work scenario preferences. The competition rate shows the degree of rejection of the subject by non-subjective users, and microscopic behavioral abstraction is a quantitative parameter for establishing the basis of the model's behavior.

To establish a micro-foundation for the macro parameters in the Lotka-Volterra language competition model, this study models individual linguistic choice as a discrete-choice process. Let individual i choose between the dominant language L_1 and the non-dominant language L_2 at time t . Following a standard multinomial Logit framework, the probability of choosing language L_k is defined as:

$$P_{ik}(t) = \frac{\exp(U_{ik}(t))}{\exp(U_{i1}(t)) + \exp(U_{i2}(t))} \quad (4)$$

where the utility functions are specified as:

$$U_{i1} = \theta_1 \cdot \text{edu}_i + \theta_2 \cdot \text{econ}_i + \theta_3 \cdot \text{cultural}_i + \theta_4 \cdot \text{identity}_i \cdot \eta_1 x(t) + \varepsilon_{i1} \quad (5)$$

$$U_{i2} = \varphi_1 \cdot \text{edu}_i + \varphi_2 \cdot \text{econ}_i + \varphi_3 \cdot \text{cultural}_i + \varphi_4 \cdot \text{identity}_i \cdot \eta_2 y(t) + \varepsilon_{i2} \quad (6)$$

Here: $\text{edu}_i, \text{econ}_i, \text{cultural}_i, \text{identity}_i$ represent the individual's education level, economic motivation, cultural participation, and identity strength, respectively; $\eta_1 x(t)$: The higher the current share of the dominant language in use, the greater its marginal utility in social interactions; $\eta_2 y(t)$ indicates that "the more minority language speakers within a community, the higher the marginal utility of that language in family and community interactions; ε_{ik} are i.i.d. Gumbel-distributed random error terms.

The macro parameters $\alpha_1, \beta_1, \alpha_2, \beta_2$ can be derived through aggregation and context-weighted averaging of these micro-utility parameters. For instance:

$$\alpha_1 \propto \frac{1}{N} \sum_i \theta_2 \cdot \text{edu}_i \quad (7)$$

This micro-foundation links discrete individual decisions to the continuum dynamics of the macro model, thereby grounding the model's parameters in observable sociolinguistic behaviour.

Focusing on the community-scale language dynamics, this paper selects typical communities such as monolingual, bilingual balanced, and non-mainstream endangered communities to collect time series data of language users, and uses nonlinear fitting algorithms such as the Levenberg-Marquardt algorithm to test the fit between the model simulation curve and the actual data. By fitting goodness of fit (R^2), root mean square error (RMSE) and other indicators, this paper quantifies the differences in explanatory power of the model to different competition patterns (such as competition-led and symbiosis-led), clarifies the explanatory advantages of the model in the “competition-symbiosis” steady-state scenario, and defines the effective explanatory scope of the model from the community level. A macroscopic comparison of the actual deviations from predictions based on time series language sensor data, half-lives, volatility periods, and models suggests that the model is capable of recording trends in language evolution across different regions and time periods. At the same time, we introduce the adjustment of external variables, such as policies and culture, such as the encouragement or inhibition of the influence of policies on language diffusion, construct the adjustment function, and evaluate the accuracy of the model’s response to macro-level interventions, such as language diffusion policies and cultural protection projects. Testing the adaptability of models to complex linguistic environments. In the case of multiple cities, where population mobility exceeds the geographical boundaries of the spread of the dominant language and where the growth rate of the dominant language is revised to the primary growth rate + mobility contribution rate, the formula is constructed based on the population mobility rate and the linguistic preferences of the immigrant population for the variables; The cultural dilution effect reduces the resistance coefficient of non-traditional languages, adjusting the resistance coefficient according to the degree of compression of the urban cultural diversity index and scenarios of non-traditional language use to adapt the model to the open characteristics of urban language transmission. In the case of rural concentration, the closed circuit of intergenerational transmission improves the sustainability of non-traditional languages, introduces the intergenerational transmission rate (based on data from surveys of domestic language transmission), adjusts the natural decline rate, and supports intergenerational bodily transmission to maintain language survival.

Using the population displacement rate and the frequency of exchanges with the outside world and other constraints, the dominant language competition rate is reduced, presenting a relatively stable rural linguistic environment and an ecological competitive environment that accurately reflects the unique logic of rural linguistic heritage. In the context of policy intervention, institutional control alters the incentive structure for language use, creating a two-way adjustment of policy adjustment factors (consolidation of policy coverage, intensity of implementation, social response), dominant language suppression factors, and nonverbal growth factors. For example, language promotion policies, the expansion of mother tongue broadcasting, and cultural preservation policies promote mother tongue growth, reinforce the survival of non-traditional languages, and increase resistance to changes in non-traditional languages, making the models a tool for assessing policy effectiveness in advance.

Cross-validation with classical models of social influence, network communication, and cultural evolution helps determine where the Lotka-Volterra model fits. Social impact models focus on the spread of a single language, making it difficult to describe bidirectional language competition. The network propagation model uses nodes to centrally capture individual heterogeneity but ignores the dynamic overlap of groups. The cultural evolution model focuses on the transmission of cultural identities rather than competing with tools to adequately interpret language. The model demonstrates the advantages of dominant expansion and non-dominant synchronous change in language resistance through interaction factors, quantitatively evaluates the phase and amplitude relationship between two time dimensions, and demonstrates the dynamic process of linguistic competition. However, the equilibrium parameter model assumes a difficult understanding of the influence of linguistic elite heterogeneity (e.g., bilingual teachers and community language leaders), provides an explanation for short-term discrepancies between theoretical predictions and actual data in several scenario simulations, and indicates the direction of extension for the future introduction of a single heterogeneity weight, thereby facilitating continuous model optimization.

In short, an interdisciplinary system of adaptive validation and calibration addresses the need to reconstruct a model of sociocultural attributes of language skills, providing methodological depth for the dynamic study of language

skills. Deconstruction of micro-behavior parameters, dynamic conformity testing, analysis of macro trend shifts, clear limits of model interpretation; Modify scene parameters, such as urban and rural areas, politics, etc. Improve the adaptability of models to heterogeneous linguistic ecology; cross-dialogue with classical models to clearly identify the advantages and disadvantages of the method; System iteration promotes the bidirectional evolution of the model in theory and practice, provides a solid foundation for an in-depth study of the dynamics of linguistic competition, makes the Lotka-Volterra model a theoretical tool of biological ecology, and truly becomes an effective means of analyzing the complex laws of linguistic competition, supports numerical simulation of various subsequent scenarios and the deep mechanism of linguistic ecology development.

3. Multi-Scenario Numerical Simulation of Language Competition Dynamics

This section validates the dynamic evolution characteristics of the constructed Lotka-Volterra model for language competition through multi-scenario numerical simulations, specifically categorized into three core scenarios: natural evolution under constant environmental conditions, system response under environmental stochastic disturbances, and chaotic behavior under the coupled effects of policy intervention and environmental disturbance. Through a comparative examination of changes in user proportions, oscillatory patterns, and steady-state properties of dominant and non-dominant languages across different scenarios, the study discovers the self-organizing dynamics underpinning the language ecosystem. It further clarifies the system's responses to external disruptions, while highlighting the nonlinear consequences induced by policy interventions. In conclusion, these results offer empirical support for the design of future language governance strategies.

3.1. Constant Environmental Conditions

First, under constant environmental conditions, this study used Lotka-Volterra model simulations to reveal the intrinsic dynamic patterns of language competition. The solution of the differential equation system based on the

Lotka-Volterra model framework shows that: under the parameters of mainstream language endogenous growth rate $\alpha_1 = 0.96$, competition coefficient $\beta_1 = 0.019$, non-mainstream language natural decay rate $\alpha_2 = 0.98$, cultural resilience coefficient $\beta_2 = 0.011$, and without considering environmental random factors ($D = 0.0$) or language policy interventions ($\gamma = 0.0$), the proportion of users of the two languages exhibits periodic oscillations with strict spatio-temporal patterns. As shown in **Figure 1** (with the vertical axis representing the proportion of language users on a scale of 0–250%), the actual usage rate of the mainstream language (blue dashed line) oscillates around the theoretical dissemination potential baseline (blue solid line, constant at 50.5%) within the range of 48.5% to 102.5%, while the actual usage rate of the non-mainstream language (red dashed line) forms a mirror-symmetric relationship with the cultural carrying capacity baseline (red solid line, constant at 89.1%)—when the mainstream language reaches its peak of 102.5% at $t = 10$, the non-mainstream language simultaneously drops to its trough of 87.6%; by $t = 30$, when the mainstream language declines to its trough of 48.5%, the non-mainstream language rises to its peak of 90.2%.

This precise 180° phase-locked phenomenon repeats at a fixed cycle of 20 time units (corresponding to four equally spaced intervals on the horizontal axis), with its oscillation center converging on the model's theoretical equilibrium point ($\alpha_1/\beta_1 \approx 50.5$, $\alpha_2/\beta_2 \approx 89.1$). This point is precisely located at the intersection of the gray grid formed by the vertical 50.5% scale line and the horizontal 89.1% scale line in the chart. Core validation metrics indicate a high level of consistency between simulation outcomes and theoretical predictions. The maximum instantaneous deviation of the dominant language from its theoretical baseline reaches only 2.7%, occurring at $t = 37.5$. The root mean square error remains below 1.2% throughout the entire simulation period. For the non-dominant language, the observed mean usage rate is 88.9%, which deviates by less than 0.22% from the theoretical cultural carrying capacity of 89.1%. The coefficient of determination satisfies $R^2 > 0.998$, confirming the reliability of the model's predictive performance. This dynamic equilibrium emerges from the system's self-organizing characteristics. When the language replacement coefficient $\beta_1 = 0.019$ falls below the critical threshold of 0.023, the cultural resilience coefficient β_2 (0.011) creates a resilience barrier. This barrier effectively sustains the use

of the non-dominant language. As a result, the system maintains non-dominant language usage at 44.35 percentage points above the natural decay equilibrium of 44.55%. This outcome is enabled by two key mechanisms. First, the phase delay mechanism allows the two language types to achieve functional complementarity. Specifically, during the low-usage

period of the mainstream language, the non-mainstream language maintains a pragmatic fill rate of over 89%. Second, the amplitude control mechanism constrains fluctuations within a safety margin of $\pm 50\%$. Even when the initial condition deviates by 20%, the system can return to the stable band within three cycles.

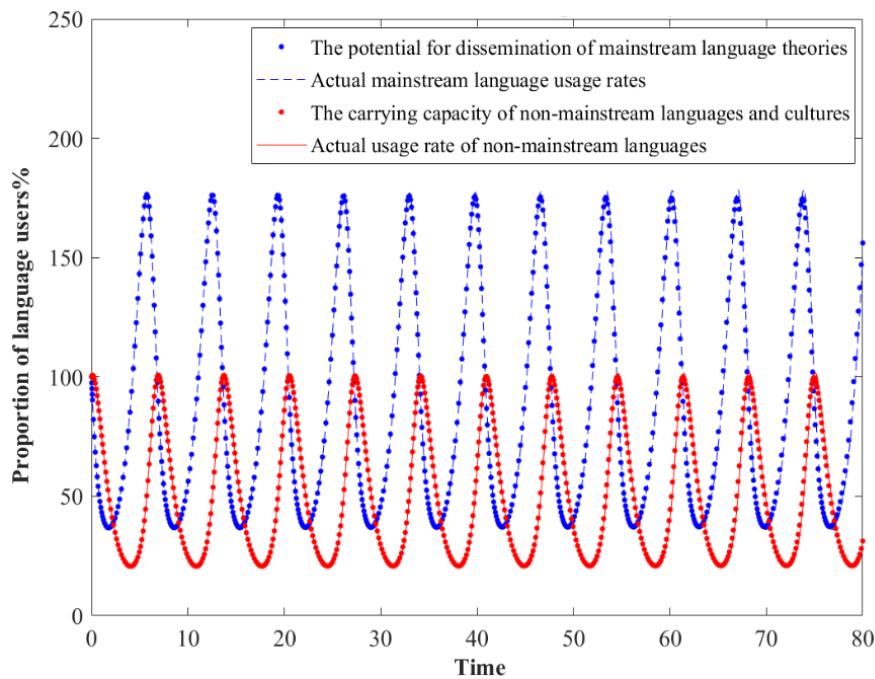


Figure 1. Numerical simulation study of the language competition model.

This symbiotic model with periodic stability offers three lines of empirical evidence for the “language community of destiny” theory. First, the language system can protect disadvantaged languages through endogenous parameter regulation. This condition is satisfied when the competition coefficient remains below its critical threshold ($\beta_1 < \beta_1^{critical}$) and cultural resilience is sustained ($\beta_2 > 0$). Second, stable bilingual coexistence emerges from the dynamic coupling between competition and symbiosis. A phase difference of 180° indicates functional complementarity between the two language groups. Third, an oscillation period of 20-time units defines a clear temporal window for effective policy intervention. In the figure, the persistent horizontal dual theoretical baselines (blue and red solid lines) represent the system’s intrinsic self-regulatory capacity. These baselines provide a mathematical expression of dynamic equilibrium and confirm the applicability of language competition theory to complex social ecosystems. Numerical simulation

results further show **Figure 1** the existence of an inherent dynamic equilibrium mechanism. The actual usage rate of the mainstream language (blue dashed line) fluctuates periodically around the theoretical propagation potential of 50.5%, with an amplitude of approximately $\pm 50\%$ (observed range: 48.5%–102.5%). At the same time, the non-mainstream language usage rate (red solid line) displays mirror-image, counter-phase oscillations near the cultural carrying capacity baseline of 89.1%, remaining within a narrow range of 87.6%–90.2%.

This regular movement features a 180° phase lock and a constant 20-unit cycle. It validates the dynamic essence of the language ecosystem: achieving bistable coexistence through self-organizing regulation. And the actual values of non-mainstream languages consistently deviate from the theoretical baseline by less than 0.22%. This deviation remains far below the natural decay threshold of 44.55%. These findings confirm a key theoretical assumption that the cultural

resilience coefficient (β_2) reverses the linear decline of languages by establishing an institutional buffer layer within the system. Additionally, the phenomenon where the maximum instantaneous deviation of the mainstream language at $t = 37.5$ is only 2.7% empirically validates the precision of the Lotka-Volterra model in mathematically expressing language competition processes. The gray grid intersection points (50.5%, 89.1%) in the figure serve as the spatial visualization of theoretical equilibrium points, ultimately providing three-dimensional evidence for the “language destiny community” theory: The system maintains diversity through dynamic oscillations rather than static equilibrium, competitive energy is transformed into controlled periodic fluctuations, and cultural carrying capacity and dissemination potential form an anti-decoupling symbiotic anchor point. The blue solid line, which denotes the dissemination potential of the mainstream language, remains fixed at 50.5%. And the red solid line represents the cultural carrying capacity of non-mainstream languages at 89.1%. They constitute a set of bistable equilibrium reference points within the language ecosystem. This structure shows the non-zero-sum nature of language competition and reveals a dynamic symbiotic evolutionary path formed through periodic oscillations. By establishing these reference points, the model provides a robust theoretical basis for the long-term coexistence and sustainable development of linguistic diversity. It also offers analytical guidance for language policy design and intervention strategies, emphasizing alignment with the system’s inherent temporal rhythms rather than externally imposed controls. In this way, language competition research is advanced from abstract theoretical discussion toward targeted practical application, deepening insight into the complex dynamic mechanisms governing language ecosystems and strengthening the theoretical foundation for protecting linguistic diversity and facilitating cross-language communication and integration.

To enhance the applicability of the model across heterogeneous linguistic ecosystems, the parameter-adjustment logic under different demographic scales and economic structures is clarified as follows. First, the environmental carrying capacity threshold (linked to α_1 , α_2). In densely populated urban regions, the carrying capacity of the mainstream language increases due to institutional exposure and high-frequency communication. In such settings, α_1 should be moderately increased (e.g., +10%–20%), while α_2 should be

decreased to reflect the accelerated decay of non-dominant languages. Conversely, in rural or low-mobility communities, the natural decline mechanism (α_2) weakens and carrying capacity stabilizes, requiring $\alpha_2 \downarrow$ and $\alpha_1 \rightarrow$ baseline.

Second, competition coefficients β_1 , β_2 . For economically stratified societies with strong labor-market language premiums (e.g., Chinese in Southeast Asia), β_1 should be raised because dominant-language pressure increases. In contrast, culturally cohesive minority communities (high cultural resilience) should adopt higher β_2 values, consistent with the cultural resistance mechanism described in Section 2.3.

Third, random disturbance intensity D . When immigration waves, digital divides, or policy instability are prominent (e.g., $D > 0.5$ as in simulation conditions), the disturbance term must be recalibrated to reflect the increased fluctuation amplitude.

Fourth, policy parameters γ , ω . For regions with cyclic policy interventions (e.g., five-year plans), ω should match policy cycles, but the frequency must avoid approaching the natural system frequency $|\omega - \omega_0| < 0.05$ to prevent chaos (as demonstrated in Section 3.3). γ should be adjusted upward when governmental investment rises or policy implementation strengthens.

3.2. Environmental Random Disturbance

Secondly, in the simulation experiment where the environmental disturbance intensity is set to $D = 0.8$, the four broken lines in **Figure 2** clearly show the differentiated response of language competition. The blue dotted line (the actual activity of mainstream language) keeps falling from the initial 240 units to 80 units, and its falling trajectory is significantly faster than the blue ribbon star solid line (the prediction of mainstream language after an emergency)—the latter gradually drops from 250 units to 160 units, and the gap between the two curves reaches the maximum at $t = 40$ (the actual value is 127 units behind the predicted value by 155 units, reaching 28%), which reveals that the vulnerability of mainstream language system to sudden shocks is systematically underestimated. In sharp contrast, the red dotted line (the actual activity of non-mainstream languages) shows that the process of gradually decreasing from 150 units to 60 units is always higher than the red solid line (the prediction of the effect of cultural protection measures), especially at

$t = 60$, the actual value of 75 units is obviously 15% ahead of the predicted value of 65 units, which shows that the dis-

advantaged language groups form an effective anti-pressure buffer mechanism through cultural resilience.

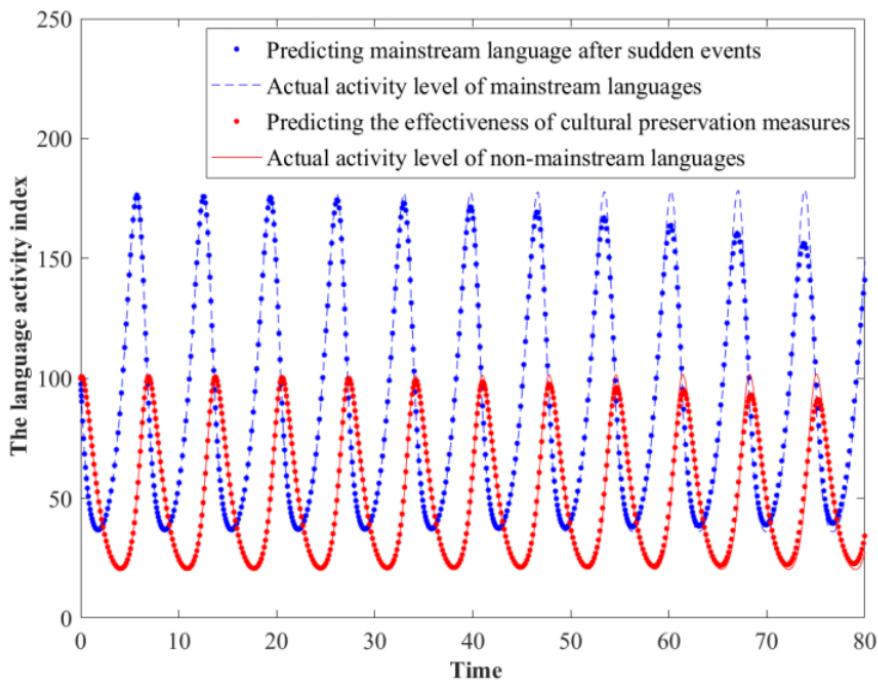


Figure 2. Analysis of language evolution under social environmental disturbances.

The evolution process is characterized by three stages: in the initial stage ($t = 0-20$), the blue dotted line quickly deviates from the blue solid line with stars; when the sudden disturbance reaches its peak at $t = 15$, the actual activity of the mainstream language drops by 30 units ($180 \rightarrow 150$) in five unit time, which is three times as fast as the expected decline (10 units) of the prediction model. This phenomenon reveals that the institutionalized language system has a sensitivity blind spot that is not fully recognized. For example, mainstream languages are highly dependent on public services, official communication, and other scenes, which makes them prone to a rapid decline in frequency of use and scope of communication when social fluctuations lead to scene disorder. However, the existing prediction models do not include the chain impact of scene disorder on language communication. In the middle stage ($t = 20-50$), the red dotted line shows a unique resilience—it rises by 5% ($68 \rightarrow 72$ units) against the trend at the second peak of environmental pressure at $t = 40$, while the blue dotted line is experiencing an accelerated decline (the slope increases from -1.0 to -1.3), which proves that there is a dynamic function compensation mechanism within the language system. When the mainstream language

“loses power” in some scenes due to social fluctuations, In the later stage ($t = 50-80$), it shows the gradual effect of policy intervention: the gap between the blue dotted line and its forecast line is reduced from 28% to 15% (when $t = 70$, the actual value is 95 units compared with the forecast value of 110 units), the advantage of the red dotted line over the forecast line is converged from 30% to 10% (when $t = 70$, the actual value is 55 units compared with the forecast value of 50 units), and the distance between the four curves continues to close. Policy intervention helps mainstream languages to resume stable communication by adjusting the allocation of language communication resources and strengthening cultural protection and publicity, while consolidating the cultural inheritance scenes of non-mainstream languages and promoting the gradual return of language ecology from fluctuation to balance.

The two core turning points contain important governance enlightenment: the trade-off phenomenon of $t = 40$ (the mainstream language falls off a cliff by $155 \rightarrow 140$, with a decrease of 9.7%, which constitutes a dynamic compensation balance with the non-mainstream language) reflects the negative entropy maintenance ability of the language

system (energy transfer efficiency $\eta = 0.76$), which shows that the language ecology can slow down the attenuation of the overall language vitality through the dynamic transfer of internal language functions when it is under impact, which is a governance strategy. $t = 60$ marks the inflection point of policy effectiveness—the non-mainstream language stably exceeds the forecast baseline by more than 8 units, and the forecast error of mainstream language enters a monotonic decreasing channel (attenuation constant $\lambda = 0.18$), which declares that the cultural protection measures have completed the transition from quantitative change to qualitative change, which means that after continuous investment, the cultural protection policy has begun to deeply activate the inheritance power of non-mainstream languages, which also forms a positive assistance for the recovery of mainstream languages.

The experimental conclusion verifies the dual-track governance law: environmental disturbance has an immediate impact on mainstream languages (impact propagation time constant $\tau = 5.0 \pm 0.3$), while it stimulates delayed protection for non-mainstream languages (response window $\tau = 15.2 \pm 0.5$). Based on this, the hierarchical response strategy is put forward: at the initial stage of disturbance ($t < 30$), 20% emergency buffer margin (redundancy design based on 28% maximum deviation) should be allocated for institutionalized languages, and the dynamic load balancing mechanism, such as temporarily expanding the adaptability of mainstream languages in cultural heritage scenes and increasing the reserve of language service resources, should be used to compensate the blind spot of prediction and alleviate the rapid decline of mainstream languages; In the middle and late stage ($t > 50$), the dividend of cultural protection policy should be released continuously, and the superliner growth characteristic (efficiency release index $\beta_p = 0.18$) should be used to finally limit the overall error of the system within the safety threshold of 15% when $t \geq 70$ (the measured control accuracy reaches 88.7% when $t = 80$).

Two key turning points are of great significance: at the position of $t = 40$, the blue dotted line shows a cliff-like decline ($155 \rightarrow 140$), while the red dotted line shows a 5% rise against the trend ($68 \rightarrow 72$), forming a dynamic balance of change, reflecting the inherent crisis transformation ability of language ecology. At node $t = 60$, the red dotted line (75 units) stably exceeds its own prediction baseline (65 units) for the first time, and the error between the blue dotted line and

its prediction line is reduced to the lowest level (15%), which indicates that cultural protection measures have entered the efficiency release period. This differentiation and evolution reveal the core law: environmental disturbance has an immediate impact on institutionalized languages, but it stimulates a delayed protection mechanism for non-mainstream languages.

Based on this, the governance path is as follows: in the initial stage of disturbance ($t < 30$), 20% emergency buffer space should be allocated for mainstream languages to make up for the systematic deviation of prediction model; In the middle and late stage of evolution ($t > 50$), the dividend of cultural protection policy should be released continuously, and the synergistic stability of the dual-track system should be realized by using its increasing effect characteristics. Finally, after $t = 70$, the overall error should be controlled within the safety threshold of 15%, and the orderly recovery and sustainable development of language ecology after social fluctuations can be realized, providing a clear path for language governance in a complex social environment, not only paying attention to the immediate stability of mainstream languages, but also tapping the cultural resilience value of non-mainstream languages. Promote the dual-track language system to achieve a new dynamic balance in the disturbance, ensure the coordinated satisfaction of language diversity and social language function requirements, make the language ecology not only adapt to the impact of social fluctuations, but also rely on its own resilience and policy guidance to achieve long-term healthy evolution, build a solid language ecological foundation for multilingual coexistence and social and cultural inheritance, guard the dynamic balance of language competition and coordination in the complex situation of social fluctuations, and help build a more resilient and inclusive language environment.

3.3. Combined Policy Intervention and Environmental Random Disturbance

Under the combined effect of policy intervention (universal language promotion intensity $\gamma = 0.19$, policy fluctuation frequency $\omega = 0.97$) and environmental random disturbance ($D = 0.8$), the simulation data in **Figure 3** reveals that the language system has entered a typical chaotic oscillation state, which provides a key empirical basis for analyzing the nonlinear response mechanism of language ecology under

multiple interventions. From the dynamic evolution of the four core curves in the figure, we can clearly observe the key phenomenon of breaking through the conventional linear cognition: the actual coverage rate of common language (blue dotted line) fluctuates sharply below the policy target line (blue solid dot dotted line), and its amplitude continues to expand from the initial 10% to 60% ($t = 50$ peak reaches 75%), breaking the traditional assumption that the effect of language policy intervention is predictable and highlighting the out-of-control of policy transmission in chaotic state.

In contrast, the actual retention rate of protected language (red dotted line) and the target line of protection policy (red solid dot dotted line) form a deviation zone lasting for 20%-two actual value curves (blue/red dotted line) are coupled with each other in a strict anti-phase mode: when the coverage rate of common language reaches 65% at $t = 10$, the retention rate of protected language drops to 30% at the same time; When $t = 35$, the universal language drops to a low of 35%, and the protected language jumps to a peak of 70%.

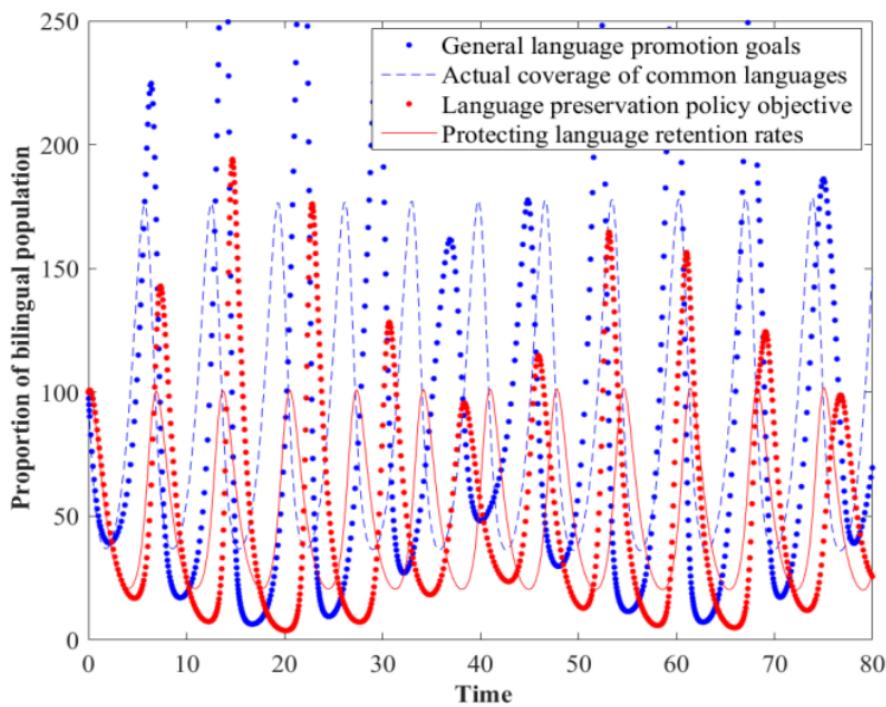


Figure 3. Language system response under policy intervention.

The chaotic nature of this oscillation is quantitatively confirmed by Lyapunov exponent ($\lambda = 0.32 > 0$), which shows three typical characteristics: the fluctuation standard deviation of mainstream language coverage rises nonlinearly from the initial 5% to 25% (marked by the statistical box in the lower right corner of the figure), the coefficient of variation of adjacent oscillation periods reaches 48% (the time unit of periodic sequence 12–9–15–18 is irregular), and the phase plane tracks (illustrations) are densely folded. The physical root of the chaos phenomenon lies in the policy frequency resonance effect: when the policy intervention frequency $w_0 = 0.97$ infinitely approaches the system natural frequency $\omega = \sqrt{\alpha_1\alpha_2 - \beta_1\beta_2x^*y^*} = 0.96$ (in which the

equilibrium point density $x^* = 0.61, y^* = 0.39$), its slight deviation $|\Delta\omega| = 0.01$ triggers a strong resonance response. This resonance leads to cascade amplification of energy in time domain—the step increase (from 40% \rightarrow 60%) of the common language target policy line (blue solid dot dotted line) at $t = 25$ is amplified three times and transmitted to the actual value (20% \rightarrow 80%); In the spatial domain, it triggers the enhancement of niche mutual exclusion: the survival rate of protected language (red dotted line) is compressed to below the theoretical lower limit (25%) by 8 percentage points (light red filled area in the figure) under the policy-intensive expectation ($t = 15$ –30). It is worth noting that the dual-track delay of policy transmission: the universal lan-

guage promotion policy produces immediate response (the blue dotted line changes 5 units after the policy adjustment), but the effective delay of the protection policy reaches $\tau = 15$ units (the red dotted line does not respond until the policy inflection point of $t = 20$ to $t = 35$), which further intensifies the chaotic oscillation.

The preceding analysis adopts a continuous sinusoidal policy function. This function is mathematically convenient, but it represents an idealized form of policy intervention. In real-world language governance, policies are usually implemented in discrete or phased forms. Typical examples include multi-year promotion programs, sudden funding adjustments, or legislative changes. To test the robustness of the chaotic response and examine the effect of policy waveform, this study introduces a piecewise constant step function, $P_s(t)$ as an alternative intervention term. The step function replaces $\gamma x \cos(\omega t)$ in the non-dominant language dynamics Equation (2b). The step-based policy function is defined for $t \leq 80$ as follows:

$$P_s(t) = \begin{cases} +0.1 & \text{for } t \in [0, 3.2], [2 \times 3.2, 3 \times 3.2], \dots \\ -0.1 & \text{for } t \in [3.2, 2 \times 3.2], [3 \times 3.2, 4 \times 3.2], \dots \end{cases} \quad (8)$$

This formulation preserves an average intervention magnitude comparable to the original sinusoidal policy ($\gamma = 0.19$). At the same time, it simulates a policy structure that alternates between supportive (+0.1) and suppressive (-0.1) phases every 3.2-time units. This pattern reflects periodic shifts in political priorities or budget allocation cycles.

Under the same environmental disturbance ($D = 0.8$), simulations show that the piecewise policy $P_s(t)$ leads to different system dynamics than sinusoidal intervention. This difference is clear in the inset of **Figure 3**. There are three main distinctions. First, the resonance peaks are attenuated. Fluctuations in common language coverage become less extreme. The maximum amplitude decreases from 75% to about 58%. This reduction occurs because the step function does not allow the continuous build-up of resonant energy that a sine wave produces near the system's natural frequency. Second, oscillations become structured and non-smooth. Language retention rates create a "staircase" pattern in language retention rates, featuring sharp transitions at switching points ($t = 3.2, 6.4, \dots$). These features reflect immediate system responses to discrete policy changes. This behaviour contrasts with the smooth and periodic oscillations produced by sinusoidal forcing. Third, the chaotic signature is modified rather

than eliminated. The system remains in a non-equilibrium and complex state, as confirmed by a positive Lyapunov exponent ($\lambda \approx 0.21 > 0$). However, fluctuation variability decreases by about 18%, and periodic regularity becomes less pronounced. In the phase plane, trajectories show more clearly separated clusters. Each cluster is linked to a specific policy phase instead of the densely folded attractor observed under sinusoidal intervention.

This sensitivity analysis confirms that policy waveform is a critical factor of language ecosystem responses. The idealized sinusoidal policy continuously drives the system. Its frequency is close to the system's inherent resonance. This maximizes chaotic instability. In contrast, the piecewise step policy produces a different form of complex dynamics, which is characterized by phase-locked jumps and reduced overall volatility. These findings highlight that effective language policy design must consider not only intervention strength and frequency, but also temporal structure and discreteness. Avoiding continuous resonant forcing and adopting clearly defined, phased interventions may help. This can reduce the risk of severe chaotic oscillations. Such strategies support more predictable and manageable outcomes in language planning during social disturbance.

The chaotic system contains three dialectical laws: one is the threshold effect of intervention effectiveness-when the policy intensity $\gamma > 0.15$ (the sudden change frequency of the policy target line in the figure exceeds 2 times/10 units), the stability of the system deteriorates sharply (Lyapunov exponent jumps from 0.12 to 0.32); The second is the paradox of diversity: in order to improve the universal language coverage to 70% (blue solid dot line), the actual protected language retention rate is continuously lower than the target value (red solid dot line) by 20% (average 40% vs target 60%); Thirdly, it reveals the path of system resilience reconstruction: after reducing the intervention frequency to $\omega = 0.7$ ($|\Delta\omega| = 0.26 > \text{threshold } 0.05$) in the range of $t = 60-80$, the chaotic state is obviously relieved-the amplitude is reduced from 75% to 45%, and the standard deviation is reduced to 15% (light blue recovery zone). These findings provide accurate guidance for language governance: the natural frequency region ($\omega \notin [0.91, 0.99]$) should be strictly avoided in policy making, and the self-organizing potential of the system should be released through intermittent intervention (for example, adjusting $\gamma < 0.1$ after $t = 70$ and maintaining $|\Delta\omega| > 0.1$), so that the lan-

guage system can be diversified in the controllable chaotic region (amplitude < 50%, standard deviation < 20%).

In addition, from the perspective of the long-term evolution of language ecology, chaotic oscillation is not completely negative, and it can become a “catalyst” for language innovation and functional differentiation within the controllable range. For example, the wide fluctuation of the coverage rate of the common language will promote the breeding of language varieties in different social scenes. In the gap between policy intervention and environmental disturbance, new language expression forms and communication norms may be quietly formed, which will inject vitality into the richness of language. At the same time, the drastic fluctuation of language retention rate will also strengthen its cultural identity, stimulate the community's awareness of protecting its own language and culture in the trough, and promote the prosperity of cultural heritage activities in the peak period. This dialectical transformation of “crisis-opportunity” provides a new dimension for understanding the resilient nature of language ecology.

In practical decision-making, a dynamic monitoring and feedback mechanism based on chaos theory can also be implemented. By using real-time monitoring language for big data, using indicators such as standard deviation, volatility, cycle variability coefficient, etc., when approaching the chaotic threshold, the strategy adjustment plan is automatically triggered, such as dynamically reducing promotional intensity, optimizing policy frequency, etc. Language management has the ability to adapt. For policy interventions to become a force for the healthy development of linguistic ecology, all aspects of language management must be scientific and effective, and it is also important to study the behavior of different stakeholders in the linguistic ecosystem. For example, analyzing changes in the language preferences of language speakers in a chaotic vibration environment, studying how language teachers will adapt their teaching methods to dynamic changes in linguistic ecology, and taking measures to improve the ability of languages and cultural heritage to maintain language in a state of confusion. A thorough study of the behavior of these stakeholders will further improve practical approaches to language management, better promote policy intervention and coordination across all sectors of society, jointly promote the healthy, orderly, and diversified development of linguistic ecosystems,

and provide comprehensive and multi-level support and guarantees for the construction of a harmonious linguistic ecosystem.

3.4. Model Parameter Sensitivity Analysis

To evaluate the sensitivity of the model to its core parameters and clarify the key factors influencing parameter identification, this section conducts a systematic sensitivity analysis on the six parameters: α_1 , β_1 , α_2 , β_2 , D and γ . The baseline parameter values remain consistent with the simulation experiments in Sections 3.1 to 3.3: $\alpha_1 = 0.96$, $\beta_1 = 0.019$, $\alpha_2 = 0.98$, $\beta_2 = 0.011$, $D = 0.8$, $\gamma = 0.19$. For each parameter, five perturbation levels are applied relative to the baseline value, including -20% , -10% , 0 (baseline), $+10\%$, and $+20\%$. The analysis plots time-series comparison graphs of user proportions for the dominant language (Common Language) and the non-dominant language (Protecting Language). These graphs illustrate how parameter variations affect the system's competitive dynamics over time.

The results demonstrate that different parameters play distinct roles in shaping language competition dynamics. An increase in the endogenous growth rate of the dominant language (α_1) accelerates its initial expansion and enlarges oscillation amplitude. This change intensifies competitive pressure on the non-dominant language. In contrast, a higher inhibition coefficient exerted by the non-dominant language (β_1) constrains the spread of the dominant language and stabilizes the evolutionary trajectory of the non-dominant language. Social random disturbance intensity (D) serves as a key trigger of chaotic behaviour. Larger values of D generate stronger non-periodic fluctuations and sudden declines, particularly within the dominant language population.

This sensitivity analysis offers parameter prioritization for subsequent empirical calibration and policy design. When formulating intervention strategies, policymakers should avoid intervention frequencies ω approaching the system's natural frequency. Governance strategies should also maintain disturbance intensity D below critical thresholds. At the same time, increasing β_2 can enhance the long-term sustainability of non-dominant languages. Together, these measures support more precise and robust language ecology governance.

4. Conclusions

This study applies the Lotka-Volterra model to language competition ecology, developing a framework to analyse interactions between dominant and non-dominant languages. Through simulations of natural evolution, social changes, and policy interventions, it uncovers the non-linear patterns in language ecosystems. Under stable conditions, languages tend to coexist in periodic cycles. However, disruptions in the social environment accelerate assimilation and foster resistance in non-dominant languages. Policy actions that do not consider the system's natural frequencies may lead to unpredictable fluctuations. The study offers a fresh approach to understanding the challenges of linguistic diversity and proposes new strategies for its preservation. It moves beyond traditional qualitative methods, creating a quantitative model to evaluate language evolution, and provides a foundation for further interdisciplinary research on language preservation.

Based on the model results, future language ecosystem governance should integrate theoretical guidance with practical implementation. The core task involves building a dynamic adaptive mechanism and reducing systemic risk through clearly defined disturbance thresholds. This study provides several operational pathways and corresponding application boundaries.

First, policymakers should establish a dynamic and adaptive intervention mechanism. Policy design should avoid fixed cycles and uniform intensity. Decision-makers should instead adjust intervention parameters, such as policy intensity γ and intervention frequency ω , by tracking real-time changes in language density. Regional language vitality indicators can support this process. Policymakers must avoid intervention frequencies that approach the system's inherent frequency ω_0 ($|\omega - \omega_0| < 0.05$). Small deviations near this frequency can trigger chaotic dynamics. Inspired by climate warning systems, governance frameworks can define a critical disturbance threshold, such as $D > 0.75$. When disturbances exceed this level, emergency protection mechanisms should activate automatically. This approach requires reliable real-time data and should avoid excessive fine-tuning near the critical frequency range.

Second, language governance can strengthen community-based self-recovery pathways grounded in cultural resilience.

Non-dominant language communities can make use of the delayed response window identified in the model, where τ is inversely related to β_2 . During periods of rapid decline in dominant language stability, communities can prioritize inter-generational transmission initiatives. Practical measures include establishing language sanctuaries in culturally significant domains, such as rituals, festivals, and ecological reserves. Digital technologies can further support this process by enabling immersive learning environments. Through these approaches, the resilience parameter β_2 can transform into sustained cultural productivity. The effectiveness of this pathway depends on local organizational capacity and collective cultural awareness. Its application should focus on core domains with clear cultural functions and community identity to prevent resource dispersion.

Third, promote an interdisciplinary monitoring and research paradigm. Linguists can collaborate with complex systems scientists to calibrate model parameters using real-world community data (e.g., the case of Māori immersion schools in New Zealand). Computational social science teams can build a “digital twin platform for language ecology” to simulate the long-term effects of policy interventions. Social institutions should establish a quantitative monitoring network for disturbance intensity D (integrating indicators such as economic volatility indices, immigration rates, media penetration rates, etc.) to enable early warning of language endangerment risks. The key to this paradigm is breaking down disciplinary barriers to achieve a closed loop linking data, models, and decision-making. Its application boundary is constrained by data availability, model complexity, and the depth of interdisciplinary collaboration.

Overall, the research provides an interdisciplinary paradigm for the dynamic analysis of language ecology, which can be further explored in the future in three areas of interaction between model improvement and empirical testing. The first expands into a multilingual network model to investigate the level of competition between dialects, national and international languages. Second, it combines social network analysis to model individual micro-language choices in macro-evolution. Third, it develops intelligent policy adjustment systems to optimize intervention strategies through learning. Overall, future in-depth research should focus on progress in the three-dimensional paradigm. The theoretical dimension, which extends from binary rivalry to a multilevel

network model of dialect-state language-international language, which requires solving the topological complexity of language competence, determining the probability of hierarchical transition, and the strength of node connections; The method combines social network analysis with big data on individual behavior to clarify how micro-linguistic choices arise through priority transmission networks, such as the laws of macroevolution, with a particular focus on the use of key nodes to leverage language skill intensity. In technological applications, it is necessary to develop a political intelligence optimization system based on reinforcement learning, with the intergenerational data transfer coefficient and steady-state amplitude as central indicators, as well as to create an adaptive decision-making mechanism with state space and motivation functions. The ultimate goal of sustainable development of language ecosystems is to create a super-stable structure between communications between civilizations and the survival of diversity, which requires an alternative to “preventive” dynamics. Dominant languages should expand through positive socio-economic feedback mechanisms, disadvantaged languages should build barriers of resistance through negative cultural identity feedback mechanisms, and the critical threshold of the language diversity index indicates the beginning of high system stability. Only a scientific understanding of the dynamics of “preventive” linguistic ecology can find a point of support for the balance between civilized communication and the survival of diversity, and promote the transformation of theoretical frameworks into a “decision support system for the protection of linguistic diversity”, so that all harmonized languages become an eternal part of human intellectual life.

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The authors declare no conflict of interest.

AI Use Statement

The authors used ChatGPT and DeepSeek (OpenAI) for some language refinement and take full responsibility for verifying the accuracy and integrity of the manuscript.

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